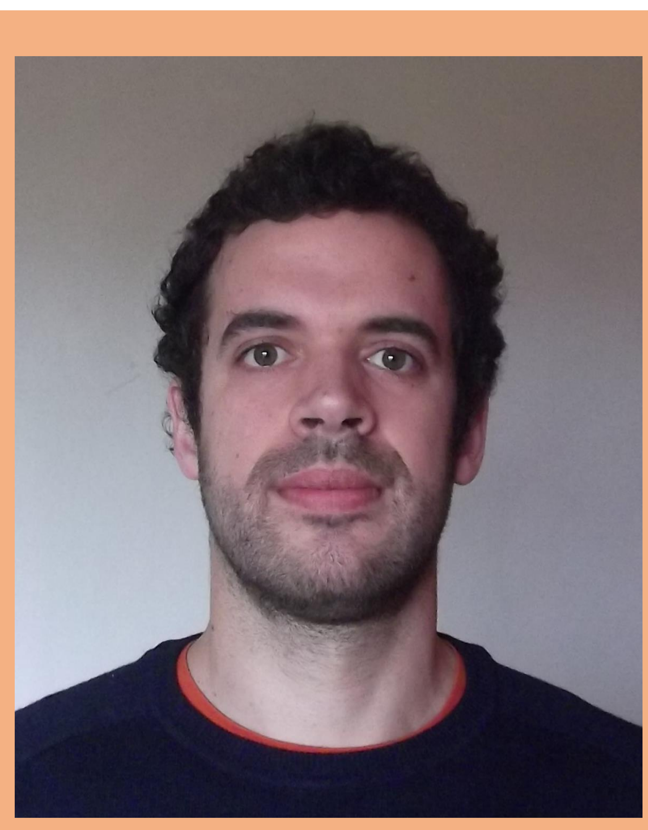
FSCOSD – Physics of Semiconductors, Optoelectronics and Disordered Systems, University of Aveiro

Solution of the explosive percolation quest

Percolation refers to the emergence of a giant connected cluster in a disordered system when the number of connections between nodes exceeds a critical value. The percolation phase transitions were believed to be continuous until recently when in a new so-called "explosive percolation" problem for a competition driven process, a discontinuous phase transition was reported. The analysis of evolution equations for this process showed however that this transition is actually continuous though with surprisingly tiny critical exponents. For a wide class of representative models, we develop a strict scaling theory of this exotic transition which provides the full set of scaling functions and critical exponents. This theory indicates the relevant order parameter and susceptibility for the problem, and explains the continuous nature of this transition and its unusual properties.



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Objectives

The common understanding that percolation phase transitions are continuous was shaken by Achlioptas et al. [*Science* **323**, 2009] that reported a discontinuous percolation transition in models whose evolution was driven by local optimisation algorithms. In work [1] we have shown that the explosive percolation transition is actually continuous. In work [2], for this explosive percolation transition in a wide set of representative models, we fulfilled the following program. We indicate the order parameter and the generalised susceptibility, find the full set of scaling relations between critical exponents, obtain the scaling functions and critical exponents, and get the upper critical dimension. In short, we develop a scaling theory of this transition.

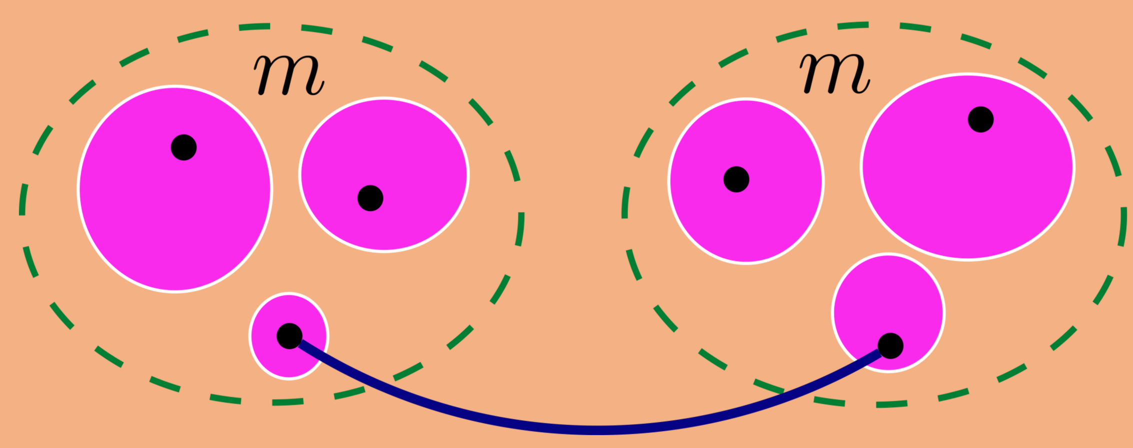
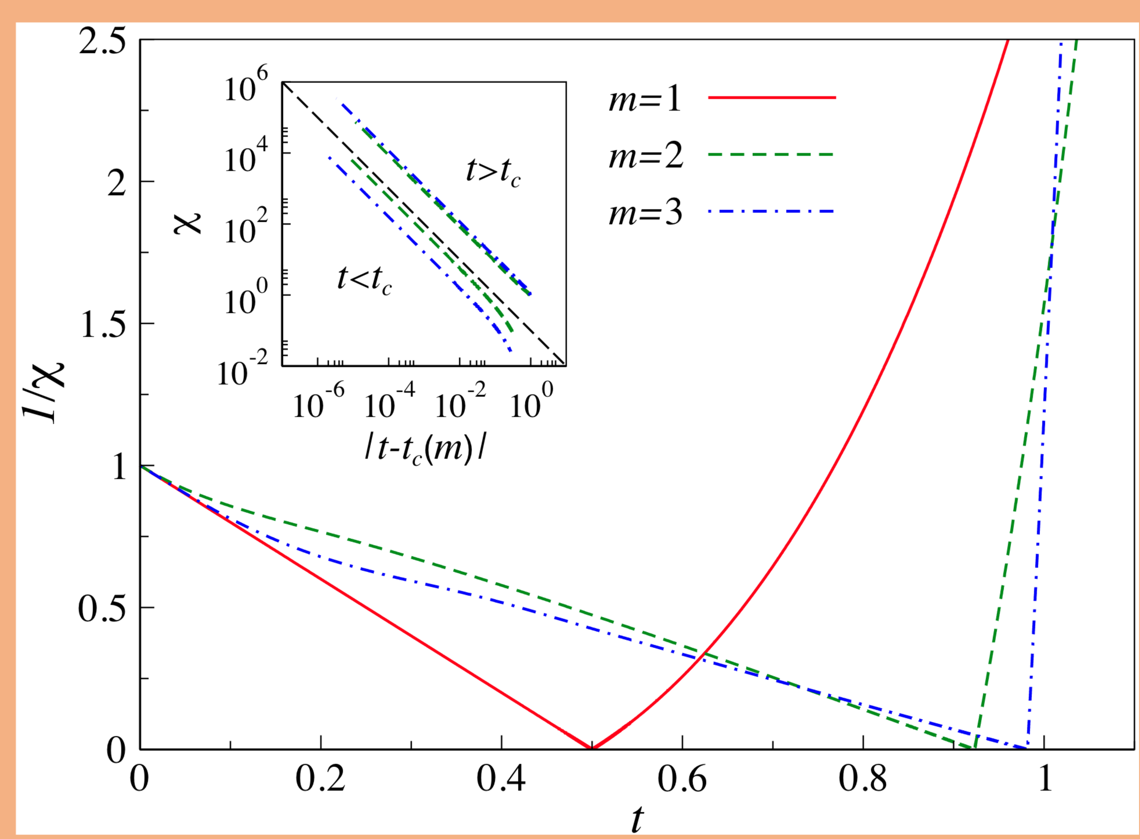


Fig. 1. At each step, two sets of m nodes are chosen at random. Within each set, the node in the smallest cluster is selected, and these two nodes are interconnected.

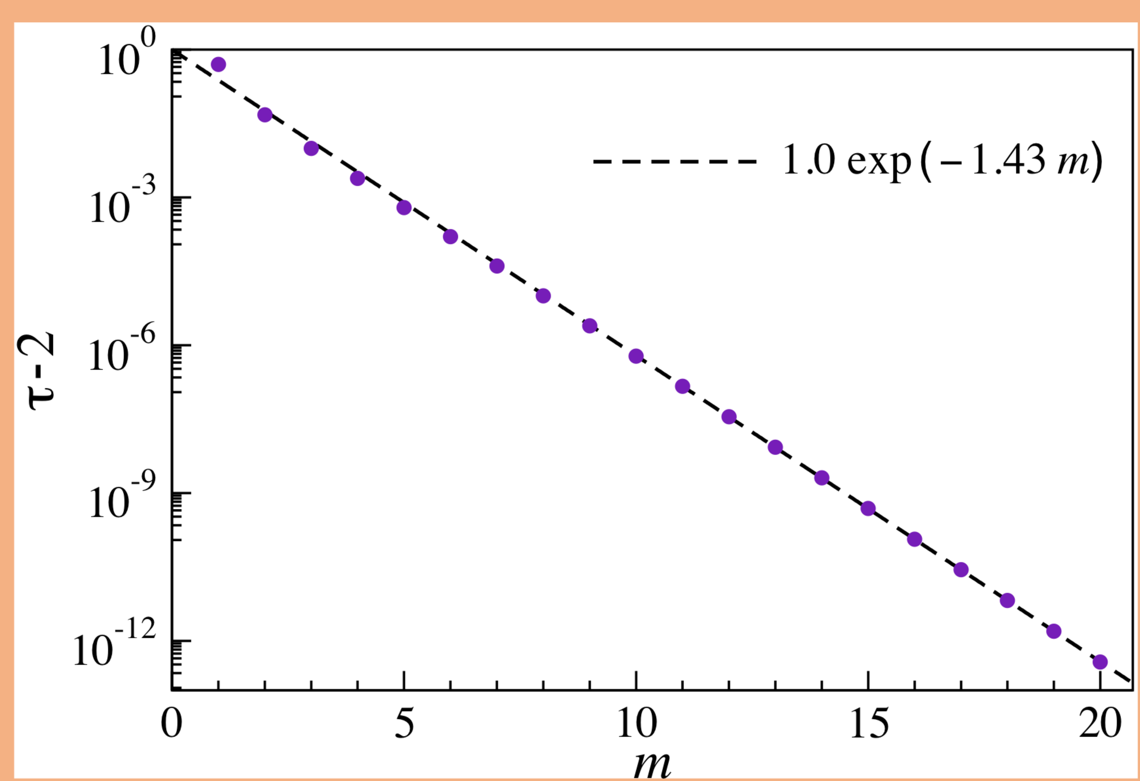
Methods and techniques

We consider a set of models that generalises ordinary percolation on classical random graphs. The number m of nodes is fixed. At each time step a new link connecting two nodes is added to the network. The evolution rules that define how these nodes must be selected are explained in Fig. 1. For the infinite system, this process is governed by a version of the Smoluchowski equation. We solve that equation in the critical region, where the scaling function of the cluster size distribution takes a scaling form. Based on the equivalence to one-state Potts model, we rigorously derived the proper order parameter and susceptibility χ for explosive percolation, which diverges according to the Curie-Weiss law $\chi \propto |t-t_c|^{-1}$.



Results

The complete scaling description which we developed explains the genuine continuous nature of the explosive percolation phase transitions. Despite of this continuity, we found and highlighted the drastic difference between ordinary and explosive percolation. We found that, unlike in ordinary percolation, the order parameter and susceptibility of explosive percolation are not the giant component size and the average of the cluster of a random node. This explains the principal novelty of critical phenomena associated with this continuous transition and its surprising features including the small values of the critical exponent of the giant component size $\beta=(\tau-2)/[1-(2m-1)(\tau-2)]$. In summary, by developing its scaling theory we explained the continuous nature of the explosive percolation transition and its unusual properties. Our work provides a conceptual and methodological basis that can be extended to new generalisations of percolation.



Publications

[1] Explosive Percolation Transition is Actually Continuous. R. A. da Costa, S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes. *Phys. Rev. Lett.* **105**, 255701 (2010)

[2] Solution of the explosive percolation quest: Scaling functions and critical exponents. R. A. da Costa, S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes. *Phys. Rev.* E **90**, 022145 (2014).

[3] Critical exponents of the explosive percolation transition. R. A. da Costa, S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes. *Phys. Rev.* E **89**, 042148 (2014).

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